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A note on T^5/Z_2 compactification of the M-theory matrix model

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Abstract

We study the T^5/Z_2 orbifold compactification of the M-theory matrix model. This model was originally studied by Dasgupta, Mukhi, and Witten. It was found that one had to add 16 5-branes to the system to make the compactification consistent. We demonstrate how this is mimicked in the matrix model.

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A recent proposal by Banks, Fischler, Shenker and Susskind (BFSS) [1] that M-theory in an infinite momentum frame is described by a certain matrix model has excited much attention and activity. This proposal has passed a number of non-trivial tests. There are indications, however, that there are certain elements still lacking in this formulation. By putting the matrix model to various tests one hopes to understand its strengths and weaknesses leading ultimately to, hopefully, the correct formulation of M-theory. The purpose of this note is to put the matrix model to another test which, in our opinion, the matrix model passes successfully. We consider compactification of the matrix theory on a five dimensional torus modded out by a Z_2 action which is supposed to be the analogue of a compactification considered by Dasgupta, Mukhi and Witten [2, 3]. This compactification is rather involved due to constraints coming from gravitational anomaly cancellation and cancellation of total charge on the compact manifold. It is of some interest then to see how these constraints arise in the matrix model compactification.

Compactification on a $d-$dimensional torus, according to the rules and explicit constructions of references [4, 5, 6, 7], results in supersymmetric Yang-Mills theory in $d+1$ dimensions (SYM_{d+1}) with 16 super-charges. For example, compactification on T^3 results in $\mathcal{N}=4$ supersymmetric Yang-Mills theory in $d=4$ [5, 6]. Toroidal compactifications always result in non-chiral gauge theories. As in ordinary string theory compactifications, one can get chiral theories by considering orbifolds of tori.

The BFSS model is $\mathcal{N}=1$ supersymmetric Yang-Mills in 10 dimensions with gauge group $\mathrm{SU}(N)$ dimensionally reduced to $d=1$. The resulting theory is matrix quantum mechanics of 9 bosonic and 16 fermionic matrices transforming in the adjoint of the $\mathrm{SU}(N)$ gauge group. The limit $N\to\infty$ is to be taken. This theory has only 16 supercharges which is the appropriate number of manifest supersymmetries in the infinite momentum frame for a theory with 32 supercharges. The Lagrangian for the theory is given by:

$$\mathcal{L}=\mathrm{Tr}\left(\frac{1}{2R}(D_0X^a)^2+\frac{R}{4}[X^a,X^b]^2+\overline{\Psi}D_0\Psi+iR\overline{\Psi}\Gamma_a[X^a,\Psi]\right) \tag{1}$$

where X^a ($a=1,\ldots,9$) and Ψ are $N\times N$ matrices. Here the Γ_a are 32×32 10-dimensional gamma matrices. Ψ is real and satisfies $\Gamma_{11}\Psi=\Psi$.

When one compactifies direction i on S^1 one essentially replaces X_i by a covariant derivative in the i direction. This rule is the reverse of the dimensional reduction procedure where one replaces a covariant derivative by the gauge field which appears as a scalar in the reduced theory with no dependence on the compactified coordinate. This logic then tells us that the compactification of the matrix model on T^5 is the same as the dimensional reduction of $\mathcal{N}=1$ SYM_{10} to $d=6$. The resulting theory is then $\mathcal{N}=(8,8)$ [1] super Yang-Mills in $d=6$ [7].

Orbifolds of matrix models have been considered recently in [8, 9, 10, 11, 12]. We are interested in a Z_2 orbifold of the above toroidal compactification where one reverses the orientation of the compactified coordinates. This is not a symmetry of the theory so one has to combine this with another action on the fields such that the combined transformation is a symmetry of the theory. We will denote the orientation reversal of the compactified directions by R and the second transformation by S. We will describe the physical interpretation of these transformations for the bosonic variables. The interpretation of the transformation of the fermionic matrices is not so clear. However, similar transformations are required for invariance of the matrix model Lagrangian, eq. (1). We will follow [8] and [11] in doubling the size of all the matrices to $2N\times 2N$ in an attempt to describe the states in the theory along with their images. All matrices can then be expressed in

[1] We shall use this notation to specify the number of real supercharges of each chirality.

the block form:

$$X = \begin{pmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{pmatrix}. \tag{2}$$

Using the notation of [11] we denote by $X_\perp$ the compactified coordinates and by $X_\parallel$ the uncompactified ones. So $X_\perp$ represents any of the matrices $X_1, \ldots, X_5$ and similarly $X_\parallel$ represents any of the matrices $X_6, \ldots, X_9$. The action of R should exchange the blocks $1 \leftrightarrow 2$ and multiply the $X_\perp$ matrices by a minus sign, leaving the rest invariant [11]:

$$\begin{aligned} R : X_\perp &\rightarrow -MX_\perp M \\ X_\parallel &\rightarrow MX_\parallel M \\ \Psi &\rightarrow \gamma_7 M\Psi M \quad , (\gamma_7 \equiv \Gamma^0\Gamma^1 \cdots \Gamma^5). \end{aligned} \tag{3}$$

where M interchanges the blocks. In [11] two different matrices M are considered: $M_1 = \sigma_1 \otimes 1_{N\times N}$ and $M_2 = \sigma_2 \otimes 1_{N\times N}$.

However, as we have stated earlier, R is not a symmetry of the matrix model. We must consider another transformation S which, along with R, is a symmetry. The reason R is not a symmetry can be understood by considering the analogous transformation in M-theory (or more precisely 11-dimensional supergravity). There R corresponds to a parity transformation. It is well known that parity alone is not a symmetry due to the presence of the Chern-Simons term. Therefore it must be combined with the transformation of the 3-form gauge potential:

$$A \rightarrow -A. \tag{4}$$

The question is to find the corresponding transformation S in the matrix model. One way to do this is to note that the transformation eq. (4) reverses the charge of a membrane. So we should look for a transformation S that changes the sign of the membrane charge in the matrix model. In [13] the supersymmetry algebra and central charges are calculated explicitly. The charge for a membrane is:

$$Z^{ab} = \frac{i}{2}\mathrm{Tr}[X^a, X^b] \tag{5}$$

If we choose the transformation S to be:

$$X_\perp \rightarrow X_\perp^{\mathrm{T}} \tag{6}$$
$$X_\parallel \rightarrow X_\parallel^{\mathrm{T}} \tag{7}$$
$$\Psi \rightarrow \Psi^{\mathrm{T}} \tag{8}$$

it is easy to see that:

$$Z^{ab} \rightarrow -Z^{ab}. \tag{9}$$

So we find, as in [11], that the complete transformation is:

$$\begin{aligned} X_\perp &\rightarrow -MX_\perp^{\mathrm{T}} M \\ X_\parallel &\rightarrow MX_\parallel^{\mathrm{T}} M \\ \Psi &\rightarrow \gamma_7 M\Psi^{\mathrm{T}} M \end{aligned} \tag{10}$$

It can easily be checked that this combined transformation is a symmetry of the matrix model Lagrangian, eq. (1) in the gauge $A_0 = 0$. In the gauge unfixed theory we must also include the transformation:

$$A_0 \rightarrow -MA_0^{\mathrm{T}} M \tag{11}$$

Toroidal compactification on T^5 then leads to 6−dimensional super Yang-Mills theory with $\mathcal{N} = (8, 8)$ supersymmetry with gauge group SU($2N$). The $X_\perp$ turn into gauge fields while the remaining $X_\parallel$ are scalars from the 6−dimensional field theory perspective. The fermion field Ψ splits into two opposite chirality complex Weyl fermions to form a Dirac fermion. All fields transform in the adjoint of the gauge group.

We can consider the action of identifying under RS on the field theory. There are only 2 distinct choices of the matrix M in the above transformation. As shown in [11] the choice of M_1 leads to a matrix model with gauge group SO($2N$) with the negative chirality fermion transforming in the adjoint representation and the positive chirality fermion along with $X_\parallel$ transforming in the second rank symmetric tensor representation. The choice M_2 leads to the gauge group USp($2N$) with the negative chirality fermion transforming in the adjoint representation and the positive chirality fermion along with $X_\parallel$ transforming in the second rank anti-symmetric tensor representation. These models have $\mathcal{N} = (8, 0)$ supersymmetry. The negative chirality fermion combines with the gauge fields (formerly $X_\perp$) to fill out a vector multiplet while the positive chirality fermion combines with the four scalars $X_{6,7,8,9}$ to form a hypermultiplet.

So compactification of M-theory has led to a six dimensional chiral gauge theory with a matter hypermultiplet. These theories have to satisfy stringent anomaly cancellation conditions as explained in [14, 15, 16, 17, 18]. The anomaly eight form for six dimensional gauge theories is

$$I_8 = \mathrm{tr}_{adj} F^4 - \sum_R n_R \mathrm{tr}_R F^4 = \alpha \mathrm{tr}_f F^4 + c(\mathrm{tr}_f F^2)^2. \tag{12}$$

In [16] it was noted that the gauge theory can be made anomaly free without introducing gravity if $\alpha = 0$ and $c > 0$ by the introduction of a tensor multiplet. If $c = 0$ then the theory is anomaly free as it stands and no additional multiplets need to be added. In the theory at hand we are considering two cases:

Case 1: G = SO($2N$), with one hypermultiplet in the second rank symmetric representation. In this case the anomaly is:

$$I_8 = -16 \mathrm{tr}_f F^4. \tag{13}$$

As noted in [18] the addition of hypermultiplets in other representations cannot set $\alpha = 0$. Thus this theory cannot be cured of anomalies by the addition of more matter or tensor multiplets.

Case 2: G = USp($2N$) with one hypermultiplet in the second rank anti-symmetric representation. The anomaly in this case is:

$$I_8 = 16 \mathrm{tr}_f F^4. \tag{14}$$

The anomaly can be cancelled completely in this case by the addition of 16 hypermultiplets in the fundamental representation (there is no need to add tensor multiplets). This has been recently noted in [18].

So we are forced to abandon Case 1, and adopt Case 2 to have a sensible theory. This case with the addition of 16 hypermultiplets in the vector representation has a very nice interpretation due to Berkooz and Douglas[19]. They modelled longitudinal M-theory five-branes in the matrix model as hypermultiplets in the fundamental representation. Thus we see that to make the matrix model compactification sensible we are forced to include 16 five-branes. This is in complete agreement with the compactification considered in [3] where one was forced to include 16 five-branes to cancel the gravitational anomaly present from the untwisted sector surviving the compactification.

To conclude, the M-theory matrix model has passed a non-trivial test. The compactification of the M-theory matrix model on T^5/Z_2 results in an anomalous gauge theory which can be fixed by the addition of hypermultiplets. These hypermultiplets are nothing but the Berkooz-Douglas

five-branes [19] in the matrix model. This parallels the low-energy considerations of [3]. We can, conversely, view this as evidence that the Berkooz-Douglas five-brane really is the longitudinal five-brane of M-theory.

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